Turbulent flows (ME625)

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Exercise problems on index notation

While converting equations into indicial notation, it will be helpful to denote coordinate axes as (x_1,x_2,x_3) in the full equations instead of (x,y,z). Similarly, use corresponding notation to denote components of a vector u=(u_1,u_2,u_3). For example, write continuity equation as

$$\frac{\partial u_1}{\partial x_1} + \frac{\partial u_2}{\partial x_2} + \frac{\partial u_3}{\partial x_3} = 0 \tag{1}$$

instead of

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \tag{2}$$

This will help you to quickly get used to indicial notation.
Write the following in tensor notation. For each equation in indicial notation, denote the free- and dummy-indices. Always write the expanded form, and introduce free- and dummy indices.

1. Dot product of two vectors. Write also the same in terms of Kronecker delta function.
2. Matrix-vector multiplication
3. Matrix-matrix multiplication
4. Cauchy equation
5. Strain rate tensor
6. Relation between shear stress tensor and rate of strain tensor
7. Navier-Stokes equations
8. Write Navier-Stokes equations and introduce Kronecker delta function in the pressure gradient term.
9. Write the following vorticity equation component-wise, and explain why 9 components of the vortex stretching term will be zero in case of a 2D flow

$$\frac{\partial \Omega_i}{\partial t} + u_j \frac{\partial \Omega_i}{\partial x_j} = \Omega_j \frac{\partial u_i}{\partial x_j} + \nu \frac{\partial^2 \Omega_i}{\partial x_j \partial x_j} \tag{3}$$

10. What is the value of δ_{ii}?
11. For velocity field (u_i) associated with incompressible flows, prove the following.

$$\frac{\partial^2 u_i}{\partial x_j \partial x_j} = \frac{\partial}{\partial x_j}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) \tag{4}$$